Letter to the Editor

Current-driven excitation of magnetic multilayers

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Abstract

A new mechanism is proposed for exciting the magnetic state of a ferromagnet. Assuming ballistic conditions and using WKB wave functions, we predict that a transfer of vectorial spin accompanies an electric current flowing perpendicular to two parallel magnetic films connected by a normal metallic spacer. This spin transfer drives motions of the two magnetization vectors within their instantaneously common plane. Consequent new mesoscopic precession and switching phenomena with potential applications are predicted.

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A magnetic multilayer (MML) is composed of alternating ferromagnetic and paramagnetic sublayers whose thicknesses usually range between 1 and 10 nm. The discovery in 1988 of giant magnetoresistance (GMR) in such multilayers stimulates much current research [1]. Although the initial reports dealt with currents flowing in the layer planes (CIP), the magnetoresistive phenomenon is known to be even stronger for currents flowing perpendicular to the plane (CPP) [2]. We predict here that the spin-polarized nature of such a perpendicular current generally creates a mutual transference of spin angular momentum between the magnetic sublayers which is manifested in their dynamic response. This response, which occurs only for CPP geometry, we propose to characterize as spin transfer. It can dominate the Larmor response to the magnetic field induced by the current when the magnetic sublayer thickness is about 1 nm and the smaller of its other two dimensions is less than $10^{-2}$ to $10^{-3}$ nm. On this mesoscopic scale, two new phenomena become possible: a steady precession driven by a constant current, and alternatively a novel form of switching driven by a pulsed current.

Other forms of current-driven magnetic response without the use of any electromagnetically induced magnetic field are already known. Reports of both theory and experiments show how the exchange effect of external current flowing through a ferromagnetic domain wall causes it to move [3]. Even closer to the present subject is the magnetic response to tunneling current in the case of the sandwich structure ferromagnet/insulator/ferromagnet (F/I/F) predicted previously [4]. Unfortunately, theoretical relations indicated that the dissipation of energy, and therefore temperature rise, needed to produce more than barely observable spin-transfer through a tunneling barrier is prohibitively large.

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However, the advent of multilayers incorporating very thin paramagnetic metallic spacers, rather than a barrier, places the realization of spin transfer in a different light. In the first place, the metallic spacer implies a low resistance and therefore low Ohmic dissipation for a given current, to which spin-transfer effects are proportional. Secondly, numerous experiments [5] and theories [6] show that the fundamental interlayer exchange coupling of RKKY type diminishes in strength and varies in sign as spacer thickness increases. Indeed, there exist experimental spacers which are thick enough (e.g. 4 nm) for the exchange coupling to be negligible even though spin relaxation is too weak to significantly diminish the GMR effect which relies on preservation of spin direction during electron transit across the spacer. Moreover, the same fact of long spin relaxation time in magnetic multilayers is illustrated on an even larger distance scale, an order of magnitude greater than the circa 10 nm electron mean free path, by spin injection experiments [7]. It follows, as we show below, that interesting current-driven spin-transfer effects are expected under laboratory conditions involving very small distance scales.

We begin with simple arguments to explain current-driven spin transfer and establish its physical scale. We then sketch a detailed treatment and summarize its results. Finally, we predict two spin-transfer phenomena: steady magnetic precession driven by a constant current and a novel form of magnetic switching.

We consider the five metallic regions represented schematically in Fig. 1. Layers A, B, and C are paramagnetic, whilst F1 and F2 are ferromagnetic. The instantaneous macroscopic vectors $\mathbf{hS}_i$ and $\mathbf{hS}_2$ forming the included angle $\theta$ represent the respective total spin momenta per unit area of the ferromagnets. Now consider a flow of electrons moving rightward through the sandwich. The works on spin injection [7] show that if the thickness of spacer B is less than the spin-diffusion length, usually at least 100 nm, then some degree of spin polarization along the instantaneous axis parallel to the vector $\mathbf{S}_1$ of local ferromagnetic polarization in F1 will be present in the electrons impinging on F2.

This leads us to consider a three-layer (B, F2, C in Fig. 1) model in which an electron with initial spin state along the direction $\mathbf{S}_1$ is incident from region B onto ferromagnet F2. Consider the moving spin-quantization frame given by orthogonal unit vectors $\hat{x}\hat{y}\hat{z}$ satisfying $\mathbf{S}_2 = S_2 \hat{z}$ and having the axis $\hat{y}$ in the direction $\mathbf{S}_2 \times \mathbf{S}_1$. This frame generally rotates adiabatically as determined by the rotating vectors $S_{1,2}(\tau)$. Using $\hat{y}$ for the axis of spin quantization in this frame, the spin state of the electron incident from region B is $(\cos \theta/2, \sin \theta/2)$. The Coulomb plus Stoner exchange potential of the magnet has the locally diagonal values $V_{\pm}(\xi)$, where $\xi$ is the position coordinate perpendicular to the multilayer plane unrelated to $\hat{x}\hat{y}\hat{z}$. The subscripts $\pm$ correspond to majority/minority-spin energy bands, respectively. Within the limitations of the WKB parabolic-band approximation, we define the $\xi$-components of the corresponding wave vectors $k_\xi(\xi)$. Employing a unit system in which $(\hbar^2/2)$ divided by the electron mass is unity, these wave numbers are given by the formula

$$k_\pm = \left( E - k_p^2 - V_\pm \right)^{1/2}$$

where $E$ is the constant energy of the electron and $k_p$ is the magnitude of the conserved component $k_p$ of the wave vector orthogonal to axis $\xi$. We let magnet F2 lie between $\xi = \xi_1$ and $\xi_2$, and place $\xi = 0$ at the center of region B. Thus we have the equality $V_+ = V_-$, and we assume $k_+ = k_-$ is real, in paramagnetic regions well outside of magnet F2 (particularly $\xi = 0$ and $\xi \gg \xi_2$). The stationary WKB Hartree–Fock spinor wave function $\psi = (\psi_+, \psi_-)$

![Fig. 1. Bottom: Coulomb plus locally diagonalized exchange potential $V_\pm$ versus position $\xi$ in a five-layer system composed of paramagnets A, B, C, and ferromagnets F1 and F2. The particle flow is rightward the charged flow leftward ($I_+ > 0$). Top: Vector diagram of spin moments $S_{1,2}$ and their current-driven velocities $\dot{S}_{1,2}$ for magnets F1,2.](image-url)
carrying unit particle flux for all $\xi \geq 0$ may be written

$$\psi(\xi) = \left( k_{-1/2}^\pm(\xi) \exp\left( i \int_0^\xi d\xi' k_+^\pm(\xi') \right) \right) \cos(\theta/2).$$

$$k_{-1/2}^\pm(\xi) \exp\left( i \int_0^\xi d\xi' k_-^\pm(\xi') \right) \sin(\theta/2).$$

The method of spin currents and momentum conservation used below is widely used in deriving the conventional exchange coupling energy written $-JS_1 \cdot S_2$ [4,9]. Moreover, in that context it is shown to be equivalent to other common methods [10].

The rightward particle flux $\Phi_-$ and the components $\Phi =$ $(\Phi_-, \Phi_+, \Phi_0)$ of rightward Pauli-spin $(= 2s)$ flux defined by

$$\Phi_-(\xi) = \Phi_+ + i\Phi_r = i \left( \frac{d\psi_+}{d\xi} \psi_+ - \psi_+ \frac{d\psi_-}{d\xi} \right)$$

satisfy general conditions of continuity. For the state (2) the Pauli-spin flux within regions B and C approaches

$$\Phi_+ = \exp\left( i \int_0^\xi (k_- - k_+) d\xi \right) \sin \theta, \Phi_r = \cos \theta$$

in the limit of slowly varying potential. These expressions describe the conical precession of one-electron spin about $S_2$ with the frequency governed by the exchange splitting $V_-$ during its passage through the magnet.

A crucial consideration is that by conservation of angular momentum the magnet reacts to the passage of one such electron by acquiring a change of classical momentum $\Delta S_2$ equal to the sum of the inward spin fluxes from both sides of magnet F2:

$$\Delta S_{2,x} = \frac{\sin \theta}{2 \cos^3 \theta/2} (1, 0, 0) = (\tan \theta/2)(1, 0, 0).$$

The total bar on electron transmission when $\hat{s}_+ = -\hat{s}_-$ causes the singularity at $\theta = \pi$ in this equation.

Briefly put, Eqs. (6) and (7) describe the complete transfer of the transverse component of incident-electron spin to the scattering ferromagnet, except for fluctuations due to geometrically-dependent wave interferences. The mean of these fluctuations will be small considering the usually broad distribution of incident-electron directions. It follows that an electric current composed of preferentially polarized incident electrons generally causes a well-defined motion of the moment of the scattering ferromagnet.

Treatment of the total electron flow in the full five-region system of Fig. 1 gives useful macroscopic expressions for current-driven spin transfer, including dynamic reactions of both magnets F1 and F2. The paramagnets A and C are considered semi-infinite. The interiors of all three paramagnets A, B, and C have the parabolic energy-momentum expression $E = k_+^2 + k_-^2 - Q^2$ where $Q$ is the Fermi vector.
and we take \( E = 0 \) to be the Fermi level. In our model, \( V_{\pm} \) generally varies with \( \xi \) only near the interfaces, so we determine \( Q \) at the center \( \xi = 0 \) of region B. For the two ferromagnets, assumed to have the same band structures but generally different thicknesses, we have \( E = k_{\pm}^2 + k_{\xi}^2 - K_{\pm}^2 \) where \( K_{\pm} \) are similarly the internal Fermi vectors for majority/minority-spin electrons, with \( K_+ > K_- \).

To treat this system, we first solve the two-component Schrödinger equation in the WKB limit for a general value of \( \theta \). Some pages of algebra are needed to do this and evaluate the fluxes (3) and (4) for these solutions. The common ballistic assumption distributes the electric current in the momentum space of paramagnet A by uniformly displacing the forward right-hand half of the Fermi sphere a constant infinitesimal amount independently of spin. If, for example, the smaller in-plane dimension of the multilayer is \( d = 100 \text{ nm} \), the ballistic condition \( \lambda \gg d \) on the mean free path \( \lambda \) will often be well satisfied at 80 K or less. At ambient temperatures, where the ballistic assumption is likely poor, the two-channel current model used in GMR theory \([1,2]\) generally introduces the polarization of the current needed for spin transfer. Integration of the fluxes (3) and (4) over occupied states provides the current densities of charge \( I_{\text{c}} \) (leftward) and spin \( I_{\text{s}} \) (rightward) as functions of \( \xi \), as in calculations of tunneling currents \([4]\) and conventional exchange \([4,9]\). By momentum conservation, the velocities of the adiabatically rotating magnet vectors are then given by

\[
\hat{S}_1 = I(\infty) - I(0), \quad \hat{S}_2 = I(0) - I(\infty)
\]  

with the notation \( \hat{x} = dx/dt \). To minimize the number of parameters in the theory, we use the ballistic assumption. In addition, we consistently average currents with respect to the phase factor \( e^{i\lambda \cdot \vec{w}} \), where \( \vec{w} \) is the thickness of spacer B.

We define the normal energy \( E_{\text{nor}} = -k_p^2 \) available to a Fermi-level incident electron for the purpose of surmounting the potential rise within one of the ferromagnets. The stationary states incident from the left (paramagnet A in Fig. 1) fall into three classes \( a, b, c \) according to the ranges of \( k_p \) defined below. The fluxes \( \Phi_a, \Phi_b \) and \( \Phi \) are identical for states belonging to a given one of these classes:

**Class** a: \( 0 \leq k_p < K_- \). Since \( E_{\text{nor}} > [\max V_{\nu}(\xi)] \) \(-K_\nu^2 \) for \( \sigma = \pm \), an electron fully transmits through the system independently of \( \sigma \). Therefore, the aggregate incident flux \( J_a \) of Class a states contributes to \( I_c \) an amount \( I_{ca} = eJ_a(\xi) \neq 0 \) and nothing \( (I_{c0} = 0) \) to \( I \) at any \( \xi \).

**Class** b: \( K_- < k_p < K_+ \). Now we have \(-K_+^2 < E_{\text{nor}} < -K_-^2 \) so both magnets transmit only electrons of polarity \( \sigma = + \) along the local axis of quantization. Those with \( \sigma = - \) are totally reflected. Using the WKB wave functions, one finds that the aggregate incident \( \sigma = + \) flux \( J_{cb} \) contributes to \( I_c \) a net charge current

\[
I_{cb} = eJ_b(4\cos^2\theta/2)/(3 + \cos \theta).
\]  

The corresponding spin currents are found to be

\[
I_b(0) = J_b[(\sin \theta)/(3 + \cos \theta)] \hat{\xi} + (I_{cb}/2e)\hat{\xi}. \\
I_b(\infty) = (I_{cb}/2e)\hat{\xi}.
\]  

**Class** c: \( K_- < K_+ < k_p \). Now we have \(-K_-^2 < -K_2^2 \) and all incident electrons totally reflect, giving \( I_{cc} = 0 \) and \( I_c = 0 \).

We combine the above results according to \( I_c = I_{ca} + I_{cb} + I_{cc} \) and \( I = I_{cb} + I_b + I_c \) and substitute the latter into the second Eq. (8) to find

\[
\hat{S}_2 = J_b(\sin \theta)/(3 + \cos \theta) \hat{\xi}. \\
\hat{S}_1 = I_{cb}(\infty) - I_{cb}(0)/2 - (I_{ca}/2e)\hat{\xi},
\]  

One similarly finds an analogous relation for \( \hat{S}_1 \). Note from the Eqs. (11) that the ratio \( \hat{S}_2/I_c \) depends on the ratio \( J_c/J_b \). Since in practice \( Q \) is often effectively nearly equal to \( K_+ \) (see below), we assume \( Q = K_+ \) when evaluating \( J_c/J_b \) under the ballistic assumption. Then this ratio becomes a function of one parameter, the polarizing factor \( P \) having the conventional definition

\[
P = \frac{n_+ - n_-}{n_+ + n_-} = \frac{K_+ - K_-}{K_+ + K_-}.
\]  

Here \( n_\pm \) are the majority/minority-state Fermi-level spin densities in the magnets. Experimental 4 K values for the ferromagnetic elements Fe, Co, Ni, and Gd are \( P = 0.40, 0.35, 0.23, 0.14 \), respectively, as obtained from tunneling between them and a superconductor \([12]\).

A result of this calculation for the five-layer system is the relation

\[
\hat{S}_{1,2} = (I_g/e)\hat{S}_{1,2} \times (\hat{S}_1 \times \hat{S}_2).
\]
where \( \hat{s}_{i,2} \) are unit vectors \( \hat{s}_i = S_i / S \). The scalar function \( g(> 0) \) is given by the formula

\[
g = \left[ -4 + (1 + P)^3 (3 + \hat{s}_1 \cdot \hat{s}_2) / 4P^{3/2} \right]^{-1}. \tag{14}
\]

The absence of any film-thickness dependences in Eqs. (13) and (14) results from our averaging the fluxes with respect to the phase of the exponent of the expression \( e^{ikw} \) occurring therein, involving phase differences across the thickness \( w \) of the spacer \( B \). These oscillations are asymptotically negligible for spacers thicker in order of magnitude than one atomic layer. This averaging operation causes the conventional exchange coefficient \( J \), determined by the spin currents present in the absence of charge current, to vanish because such oscillations of \( J \) with spacer thickness are symmetric about zero in mean-field calculations [6,9,10]. Only the current-driven coupling represented by Eq. (9) remains after consistent application of this averaging procedure. The practical consequence is that the predicted dynamics will not be greatly diminished even when the presence of atomic steps in the crystalline interfaces tends to nullify \( J \) This circumstance is analogous to the persistence, both in theory and experiment, of GMR even under such conditions that large spacer thickness or rough interfaces make \( J \) negligible. Moreover, current-driven spin transfer is insensitive to spacer thicknesses smaller than the mean free path (10–30 nm at ambient temperature).

Noteworthy is the prediction of Eqs. (13) and (14) that the five-layer dynamics are reversible with respect to sign of the electric current. It is the subsequent spin transfer back to the polarizing magnet by reflected minority electrons discussed before Eq. (7) which causes the polarizing magnet to react dynamically. Note also the equality \( |\hat{S}_1| = |\hat{S}_2| \), even though the magnets may differ in thickness. (For \( P < 1 \), it may be special to our choice of identical magnet parameters.) The magnitude of these velocity vectors is plotted in Fig. 2.

Note that the spin transfer predicted by Eqs. (13) and (14) for \( P < 1 \) generally vanishes at \( \theta = 0 \) and \( \pi \) as in the case of Eq. (6) based on our three-layer discussion. Its functional dependence on \( \theta \) tends to that of Eq. (7) in the limit \( P \rightarrow 1 \). However, its magnitude is just half of that inferable from Eq. (7) because of the multiple minority-spin reflections tending to confine electrons within the ‘quantum well’ defined by the spacer, and thus share the spin transfer among the magnets \( F_1 \) and \( F_2 \).

We leave to the reader the immediate geometric proof of the relation \( |\hat{S}_{1,2}| = |I_c/2e| \tan(\theta/2) \), holding for \( P = 1 \), from the conservation relation \( \hat{S}_1 + \hat{S}_2 = (0) - (0) = (I_c/2e)(\hat{s}_1 - \hat{s}_2) \) and the assumption that the vectors \( \hat{s}_{i,2} \) lie within the plane common to \( S_1 \) and \( S_2 \). This relation does not rest on the WKB or other approximations, but is a logical consequence of the ‘perfect spin polarizer’ concept.

The geometric relationships between these velocities dictated by the vector products in Eq. (13) are illustrated by the vector diagram in Fig. 1. The counter-intuitive tendency for the magnetic moments to rotate in the same direction propeller fashion is made consistent with angular-momentum conservation when the spin currents in regions A and C are considered, viz. Eqs. (8). These motions of \( S_1 \) and \( S_2 \) within their common plane contrast with the orthogonal precessions like \( \hat{S}_1 \parallel \hbar J \hat{S}_1 \times \hat{S}_2 \) dictated by the conventional exchange Hamiltonian \( -J \hat{S}_1 \cdot \hat{S}_2 \). It is this new property of current-driven exchange which implies the novel mesoscopic magneto-dynamics illustrated below. These motions due to spin transfer can dominate over those due to precession about the magnetic field \( H = I_c d / 2 \) circulating about the current when the smaller in-plane dimension \( d \) satisfies the order-of-magnitude inequality \( d < 1 \mu m \times (10^3 \ G/\mu S) \times (1 \ nm/\mu m) \) where \( \mu m \) is the magnetic film thickness.
One defect of the WKB approximation is that it allows only perfect transmission or perfect reflection of the majority- or minority-spin component at the normal/magnet interface, not allowing the coefficients to range between 0 and 1. A measure of the severity of this defect is provided by first-principles calculations of transmission probability, which we define as $T_{kp}$ with $kp$ now the transverse component of crystalline momentum, for Bloch electrons crossing the interface from a paramagnet into the majority-spin band of a ferromagnet [8]. In favorable cases, Fig. 1 of this reference shows results satisfying $T_+ > 0.96$ within the major central portion of the space of $k_p$ describing the Fermi surface. Toward the edge of this space, $T_+$ decreases steeply down to $T_+ < 0.04$ within a narrow fringe. The mean transmissions over the Fermi surfaces in these favorable cases have the values $T_+ = 0.86$ (Ag/Fe 001), 0.84 (Au/Fe 001), 0.79 (Cu/Co 001), 0.75 (Cu/Co 111), 0.66 (Cu/Co 110). These features qualitatively resemble our WKB picture with $Q$ a little greater than $K_+$, for which $T_+(k_p) = 1$ in the main region $k_p < K_+$ and $T_+(k_p) = 0$ in the smaller annular fringe region $K_+ < k_p < Q$. The case Cr/Fe(001), however, is rather unfavorable in this respect because $T_+(k_p)$ differs considerably from both 0 and 1 on most of the Fermi surface [8].

Therefore, our use of the WKB approximation is reasonable for certain compositions including the favorable cases incorporating the noble-metal spacers Ag, Au, and Cu mentioned above. Our assumption of spherical Fermi surfaces satisfying $Q = K_+$, used to derive Eqs. (13) and (14), gives $T_+ = 1$ for all $k_p$. Therefore, it is a special case not far from the facts indicated by the differences between the computed $T_+$ values, quoted above, and 1. Also, from the general smallness of transmission $T_-$ into minority bands computed [8] for Ag/Fe ($T_- = 0.16$) and for Au/Fe ($T_- = 0.17$), we predict the peaking of the spin-transfer rate at large $\theta$ shown by the curves in Fig. 2 with large values of $P$ and reflected by the limiting Eq. (7).

To obtain an idea of the remarkable new phenomena made possible by current-driven spin transfer, consider an effective uniaxial anisotropy field $H_a$, which includes the effect of magnet shape, and Gilbert damping coefficient $\alpha$. The Landau–Lifshits equation for such a single magnetic domain, modified to include the term (13), is

$$\dot{S}_z = \dot{S}_x \times \left( \gamma H_c e \cdot c - \alpha \dot{S}_z + e^{-1} \frac{g_s}{g_s} \times \dot{S}_z \right),$$

(15)

where $\gamma$ is the gyromagnetic ratio and a fixed frame is defined by orthogonal unit vectors $a$, $b$, $c$ of which $c$ is the symmetry axis of anisotropy. For the sake of illustration, we assume $S_x$ is constant in time because F1 is much thicker than F2 or has a much larger damping, etc. In addition, we align $S_x$ with the anisotropy axis of $S_z (\dot{S}_x = c)$.

The motion of magnet F2 is found by substituting the solution

$$\dot{S}_x = (\sin \theta)(a \cos \omega t + b \sin \omega t) + c \cos \theta$$

(16)

into Eq. (15) under the assumption of weak damping. Applying the condition $|\dot{\theta}| \ll |\omega|$ yields the lowest-order relations

$$\theta = -\left( \alpha \gamma H_a \cos \theta + I_e g/S_x c \right) \sin \theta,$$

(17)

where the function $g(\theta) > 0$ is given by Eq. (14) with $\dot{s}_1 \cdot \dot{s}_2 = \cos \theta$.

Qualitatively different behaviors of magnet F2 occur, depending on the sign of $H_a$. A steady precession with constant polar angle $\theta$ may occur for a constant current under the condition $H_a < 0$ making $+\epsilon$ the hard directions of magnetization. Another condition is that the quantity within parentheses in Eq. (17) vanishes for a value of $\theta$ different than 0 and $\pi$. Thus the frequency $\nu$ is tunable according to $2\pi \nu = \omega = I_e g/e\alpha S_2$. Reasonable material parameters can provide $\nu = 10$ GHz using $I_e = 10^9$ A cm$^{-2}$, which implies the feasibility of a monodomain sub-micron-scaled microwave-frequency oscillator powered and tuned by a constant applied current.

In the case $H_a > 0$, $\pm \epsilon$ are easy directions for $S_x$. Under some conditions, time-dependent solutions of Eq. (15) describe switching with $\theta(t)$ varying between orientations near the easy directions $\theta = 0$ and $\pi$. Switching away from $\theta = 0$ is subject to the threshold condition $I_e < -eS_2 \alpha \gamma H_a/g(0)$ obtained by means of small-$\theta$ expansion. Switching away from $\theta = \pi$ is governed by $I_e > eS_2 \alpha \gamma H_a/g(\pi)$ for $P < 1$. For reasonable material parameter values, repetitive switching by alternating 1 ns wide pulses of applied current density on the order of $10^7$ A
cm$^{-2}$ is predicted. This effect may have applications to high-speed, high-density digital storage and memory.

One general advantage of devices based on current-driven spin transfer is the utter modesty of current supply demanded by the all-metal structure. For a device diameter $d$, the Landauer-type ballistic resistance [13] is of order $R \approx 2\pi e^2 k_B T_{ch}^{-2} d^{-2}$, where $T_{ch}$ is a characteristic overall transmission coefficient for the layer system $/F1/B/F2/$. Our preferred value $d = 100$ nm gives $R \approx 0.2 T_{ch}^{-1}$ $\Omega$. Therefore the requirement that the lead resistance plus the internal resistance of the current generator exceed $R$ is modest indeed with one exceptional case (in need of clarification) that $P$ is near 1 and $\theta$ is near $\pi$. For then, $T_{ch}$ approaches 0 and $R$ is singular, totally blocking currents. The power dissipated in the multilayer for $j = 10^7$ A cm$^{-2}$ is only $0.2 T_{ch}^{-1}$ $\mu$W. When the effective anisotropy field opposing switching exceeds the order of $10^4$ Oe, conservation of energy will require upward revision of these estimates, but this subject is beyond the scope of the present article.

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